Answer **any nine** questions: $9 \times 5 = 45$

1. If $y = (x^2 - 1)^2$, then prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

2. Determine the value of $'a'$ such that $\lim_{x \to 0} \frac{a \sin x - \sin 2x}{\tan^3 x}$ exists and equal to 1.

3. Find the maximum value and minimum value of $\cos x + \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x$ in $(0, \pi)$.

4. Examine the convergence of the series $\frac{5}{1.2.4} + \frac{7}{2.3.5} + \frac{9}{3.4.6} + \frac{11}{4.5.7} + \ldots \ldots$

5. Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n}{n^2+1}$.

6. Find the radius of curvature of the curve $x^3 + y^3 = 3xy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

7. Find the envelope of the family of straight lines $x \cos \alpha + y \sin \alpha = 4$ ( $\alpha$ is the parameter) .

8. Find the asymptote of the curve: $x = \frac{t^2+1}{t^2-1}, y = \frac{t^2}{t-1}$.

9. Find the differential equation of the family of circles touching the axis of $y$.

10. State and prove D'Alembert's ratio test for convergence of sequence of positive real numbers.

11. State and prove Cauchy's theorem on limits.

12. Discuss the convergence of $p$-series.

13. Show that the sequence $\left\{\left(1 + \frac{1}{n}\right)^n\right\}$ is convergent.